Supersonic Jet Penetration (up to Mach 4) into a Mach 2 Airstream

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The effect of injection Mach number on jet penetration was determined from concentration measurements. Helium was injected transversely to the free stream from a flat plate at injection Mach numbers of 1, 2.4, 2.7, 3.5, and 4.0 and various total pressures. Jet penetration was found to increase with injection Mach number, the penetration dependence on Mach number increasing with downstream distance. For equal mass flows, supersonic injection resulted in as much as a 25% increase in penetration over that obtained with sonic injection. Existing correlations were found to be inadequate to fully describe the data. Penetration varied with jet to free-stream momentum ratio to the one-half power but only for a given Mach number and downstream position. Dividing the penetration by mass flow correlated the data for all Mach numbers for a given downstream position only. An expression based on momentum, Mach number, and downstream distance was developed which correlated both the present data and a set of data found in the literature.

Nomenclature

A,d = nozzle area and diameter, respectively
M = Mach number
$\dot{m}$ = mass flow
P_0 = total pressure
q = dynamic pressure, $\rho v^2/2$
v = velocity
x = downstream distance from nozzle centerline
y = penetration (1% He concentration point) measured normal to plate surface
α = angle of injection with horizontal
γ = specific heat ratio
ρ = density

Subscripts

a,e,j = free stream, exit, and jet, respectively
s = supersonic

Superscripts

$*$ = throat
$'$ = sonic

Introduction

THE fuel distribution requirements of supersonic combustion ramjets give rise to the study of secondary jet penetration in a supersonic mainstream. The high momentum of the supersonic airstream causes the injected fuel to be turned rapidly downstream, limiting mixing and combustion efficiency. Injection schemes which increase fuel penetration without increasing the combustor total pressure loss would be highly desirable for efficient engine design. One injection technique that has been considered previously[1–3] and is the subject of this study is that of normal injection at supersonic velocity.

A complex pressure field and shock pattern are associated with a jet emerging into a supersonic airstream. Injecting at supersonic velocity may alter the pressure and shock structure somewhat and therefore affect the penetration and spreading of the injected gas. Also, the momentum of a supersonic jet is changed relative to that of a sonic jet of equal mass flow and total pressure. Momentum has a maximum value at $M_j = (2)^{1/2}$.

Existing penetration data are available only over a limited Mach number range, up to Mach 2.2, and are difficult to compare because of inconsistent definitions of penetration. Schetz et al.[2], measured the position of the Mach disk (a shock feature of an underexpanded jet) and showed an increase in penetration with jet Mach number up to about Mach 2 where a local plateau was reached. Orth and Funk[3] used both Mach disk position and concentration measurements to characterize the jet penetration. From their concentration measurements they obtained about a 10–15% increase in penetration when comparing supersonic to sonic injection. Vranos and Nolan[4] determined the jet boundary using concentration measurements and concluded that penetration normalized by exit diameter decreased with injection Mach number.

In this study helium was injected at various pressures from a flat plate mounted in a Mach 2 wind tunnel. Injector inserts were used to give nominal injection Mach numbers of 1, 2.4, 2.7, 3.5, and 4.0. Concentration measurements were used to characterize the jet penetration. Comparisons were made with available correlations in the literature. The present data and the data of Orth and Funk[3] were correlated with a newly developed expression based on momentum, Mach number, and downstream distance.

Apparatus and Procedure

Model and Tunnel

The flat-plate model was mounted through a side wall of a 10 × 3.84-in. (25.4 × 9.75-cm) wind tunnel[6,7] as shown in Fig. 1. A 0.125-in. (0.318-cm) passage was provided through the model up to the injector insert. The injection nozzles were located 2.29 in. (5.82 cm) from the leading edge of the plate on the tunnel centerline. Injector inserts were threaded flush with the top surface of the plate as shown in the cross section in Fig. 2 for the sonic nozzle. A pressure-tight seal was obtained by O-ring compression. All nozzles had the same external dimensions and internal radii, and throat diameters of 0.080 ± 0.001 in. (0.2032 ± 0.0025 cm). The total divergence angle was held constant at 17° for all supersonic nozzles. The supersonic nozzles are shown in the insets in Fig. 3.

Presented as Paper 70-92 at the AIAA 8th Aerospace Sciences Meeting, New York, January 19–21, 1970; submitted February 16, 1970; revision received April 27, 1970.

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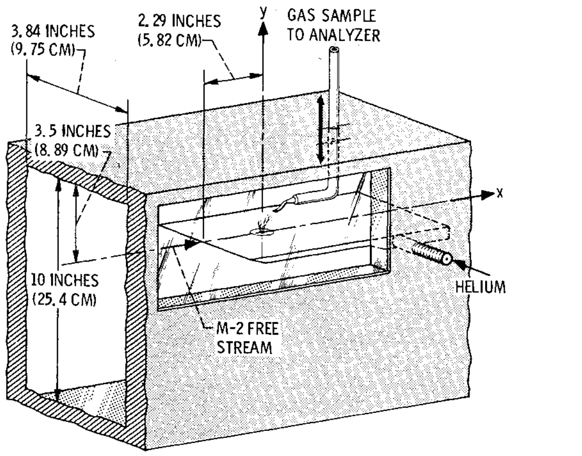

Fig. 1 Flat plate model installed in supersonic tunnel.

The Mach 2 airstream had a total pressure of 13.54 psia (93.4 KN/m^2) and a total temperature of 627°R (348°K). The Reynolds number at the injection location was 5.35 × 10^5 and the turbulent boundary-layer thickness was calculated to be 0.061 in. (0.155 cm).

Nozzle Calibration and Helium Detection

Helium was used to calibrate the supersonic nozzles for exit Mach number into still air. Hypodermic tubes of 0.020 in. (0.051 cm) o.d. were used to measure the Pitot pressure at the nozzle exit and the total pressure in the model passage. The exit Mach number was obtained from the pitot to total pressure ratio and the compressible flow tables. The calibration curves are shown as a function of injection total pressure for three ambient back pressures in Fig. 3. The Mach numbers calculated from the measured throat and exit diameters differed from those obtained from the pressure measurements by less than 10%. For all nozzles, as the ambient pressure was lowered, the nozzle flow separated at lower total pressures.

When helium was injected into the supersonic airstream rather than into still air, it was no longer obvious what back pressure the jet experienced because of the resulting complex flow situation. The freestream static pressure, 1.7 psia (11.7 KN/m^2), could be used to define the back pressure, and all nozzles would flow full for the total pressures used to obtain penetration data (noted by the large ticks on the abscissa scales in Fig. 3). A second back pressure which could possibly control nozzle flow, was suggested in Ref. 5 as 0.8 of the static pressure behind a normal shock in the freestream. For the tunnel conditions of this study, this second definition of back pressure was 6.2 psia (43 KN/m^2). If 6.2 psia is in fact the pressure to use to determine if nozzle flow separation occurred, it indicates that two of the total pressures used at M_j = 3.99 could result in separated nozzle flow and that the penetration data at these conditions might have to be interpreted with some caution.

The helium detection system was the same as that used in Refs. 6 and 7. A small boundary-layer probe (Fig. 1) was mounted through the top wall of the tunnel and set at

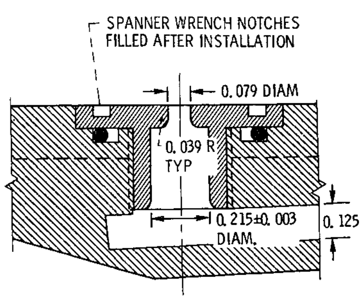

Fig. 2 Cross-sectional view of sonic nozzle installed in flat plate. (All dimensions in in.).

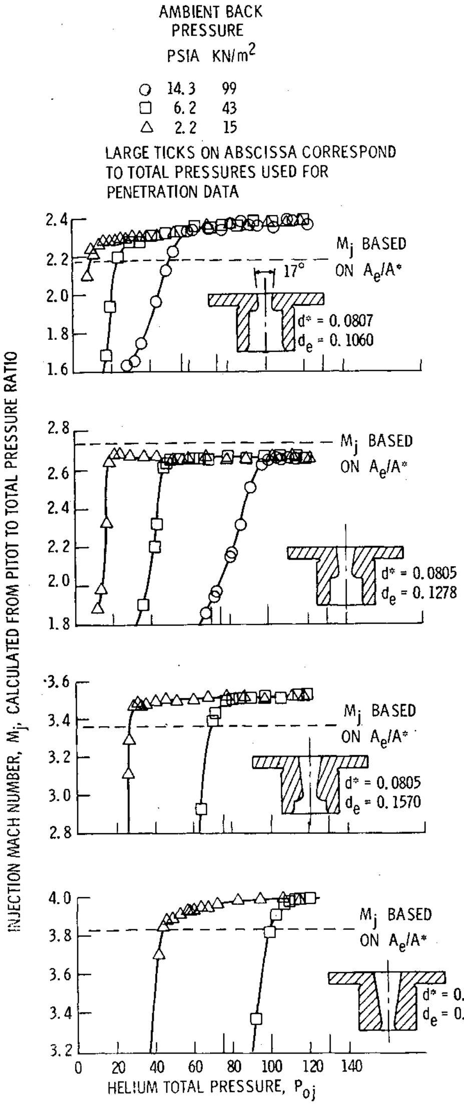

Fig. 3 Calibration of supersonic nozzles. (Dimensions in inches.)

x/d^* positions of 4.1, 10.1, and 16.7. Samples of the helium-air mixture were fed into an on-line spectrometer tube and the resulting helium volume concentration output displayed on an x-y plotter. The complete set of test conditions are presented in Table 1.

Results

Typical helium concentration data obtained in this study are shown in Fig. 4 for sonic injection at x/d^* = 4.1. Complete concentration profiles down to the plate surface were not obtained. The vertical distance at which a specific

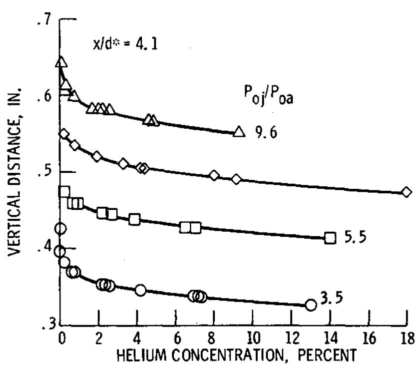

Fig. 4 Typical concentration data for sonic injection.

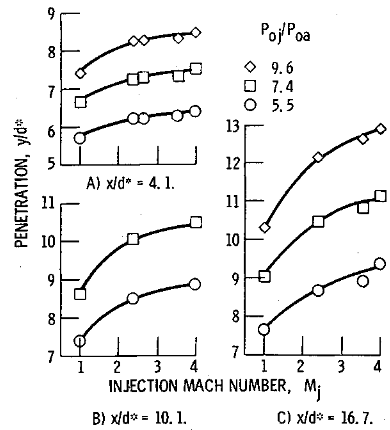

Fig. 5 Effect of injection Mach number on penetration.

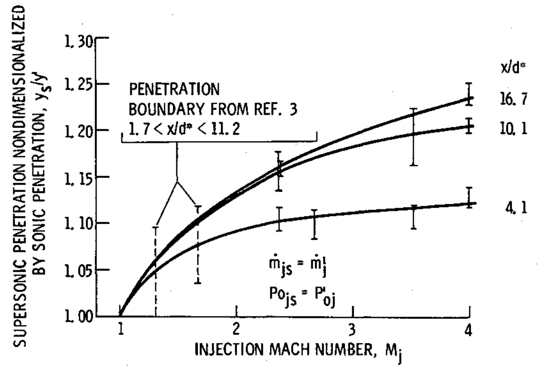

Fig. 6 Increase in supersonic over sonic penetration as a function of injection Mach number.

helium concentration was observed increased with increasing injection total pressure (P_{oa} = constant). As the distance above the plate increased, the helium concentration decreased until helium was no longer detected. Since the vertical distance at which helium concentration dropped to zero was found to be rather difficult to establish,[7] the 1% He boundary was chosen in this study as a representative measure and definition of the penetration. Considering the errors in positioning the probe, measuring concentration, and fairing the curves (Fig. 4), the total precision in estimating this y was found to be ±0.010 in. (±0.125 on y/d^*).

At the first downstream station, $x/d^* = 4.1$ (Fig. 5a), a small increase in penetration was found with increasing injection Mach number at three different pressure levels. Most of the increase occurred between Mach 1 and Mach 2.5. A maximum increase of 13% occurred when Mach number was increased from 1 to 4 at $P_{oj}/P_{oa} = 9.6$. At the larger x/d^* positions (Fig. 5b and c) the effect of Mach number on penetration increased. The greatest increase in supersonic over sonic penetration of 25% occurred at $x/d^* = 16.7$ and $P_{oj}/P_{oa} = 9.6$. Because the throat diameter was held fixed in this study, the jet exit pressure decreased with increasing M_j for equal mass flow $\dot{m}_j$ and total pressure P_{oj}; thus, the jet was underexpanded at $M_j = 1$ and overexpanded at some higher Mach number. Although the data do not indicate any unusual behavior, the circular and square data points at $M_j = 3.99$ are those for which there may be some question of nozzle flow separation, depending on the way in which back pressure is determined.

Measurements of the shock structure or the pressure field have not been made in this study. However, the limited influence of M_j may be attributable to the fact that the position of jet momentum loss (i.e., Mach disk location) does not vary appreciably as M_j is increased.[2,3]

Table 1 Test conditions and results [Constants: $M_a = 2$, $d^* = 0.080 \pm 0.001$ in. (0.2032 ± 0.0025 cm), $P_{oa} = 13.54$ psia (93.4 KN/m^2)]

M_j	x/d^*	P_{oj}/P_{oa}	q_j/q_a	y/d^*
1.0	4.1	3.54	4.02	4.56
		5.53	6.28	5.70
		7.38	8.37	6.64
		9.60	10.9	7.40
	10.1	5.53	6.28	7.38
		7.38	8.37	8.60
	16.7	5.53	6.28	7.66
		7.38	8.37	9.03
		9.60	10.9	10.30
2.37	4.1	4.13	3.86	5.31
		5.53	5.17	6.23
		7.38	6.89	7.25
		9.60	8.96	8.26
	10.1	5.53	5.17	8.49
		7.38	6.89	10.04
	16.7	5.53	5.17	8.70
		7.38	6.89	10.48
		9.60	8.96	12.13
2.67	4.1	5.16	4.12	5.91
		5.53	4.42	6.18
		7.38	5.89	7.28
		9.60	7.66	8.25
3.52	4.1	5.53	2.66	6.25
		7.38	3.55	7.30
		9.60	4.61	8.29
	16.7	5.53	2.66	8.91
		7.38	3.55	10.81
		9.60	4.61	12.61
3.99	4.1	5.53	2.05	6.38
		7.38	2.73	7.49
		9.60	3.55	8.44
	10.1	5.53	2.05	8.84
		7.38	2.73	10.44
	16.7	5.53	2.05	9.40
		7.38	2.73	11.15
		9.60	3.55	12.90

Comparisons with Previous Work

The supersonic hydrogen penetration data of Orth and Funk, obtained from concentration profiles in Ref. 3, and the results of this study are shown in Fig. 6. Supersonic pene-

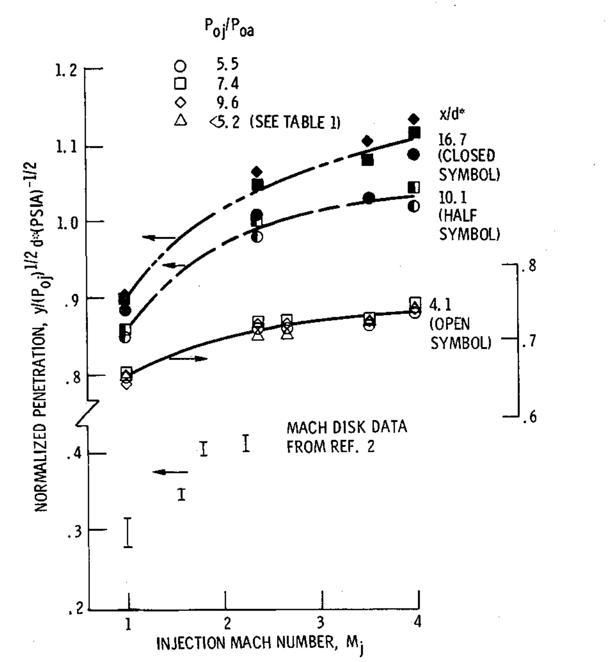

Fig. 7 Penetration normalized by mass flow term as a function of injection Mach number.

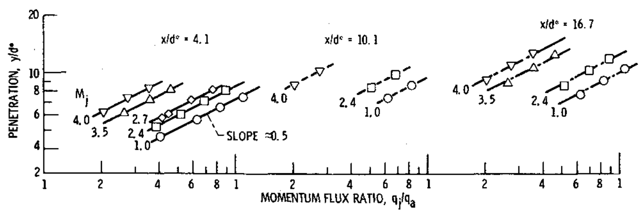

Fig. 8 Variation of penetration with momentum flux ratio.

tration y_s was divided by sonic penetration y' for equal $\dot{m}_j$ and P_{oj}. This penetration ratio increased with M_j for both sets of data. An x/d^* effect on y_s/y' was apparent for the present data whereas the data of Ref. 3 did not show a consistent trend of y_s/y' with x/d^*. Orth and Funk have compared their supersonic penetration data to a prediction of the effect of Mach number on penetration,[1] which applies for a matched pressure condition between jet exit and freestream. Matched pressure was not maintained in the present study.

Schetz et al.,[2] have proposed a correlation of penetration normalized by mass flow. They showed that the Mach disk location correlated with $y/(P_{oj})^{1/2}d^*$ as a function of injection Mach number and concluded that penetration reached a local plateau at $M_j = 2$. Their data and the present data are shown in Fig. 7. A reasonably good correlation was obtained at any particular x/d^* position. The Mach disk occurred at x/d^* on the order of 1–2. At x/d^* values of 10.1 and 16.7 there was a slight pressure effect still evident after the normalization which indicated that the $(P_{oj})^{1/2}$ factor was not exact for the higher x/d^* values.

Torrence[9] has shown that sonic penetration data correlate roughly with dynamic pressure ratio (or momentum flux ratio), q_j/q_a. However, it was also shown that at $q_j/q_a \gtrsim 5$ there was a definite dependence of penetration on downstream distance. The present data are shown as a function of q_j/q_a in Fig. 8; note the repeated abscissa scale. For any given M_j and x/d^*, penetration correlated well with momentum flux ratio with a slope 0.5.

For injection into a subsonic stream, Abramovich[8] presented correlations of the form

$$y/d^* = (q_j/q_a)^m(x/d^*)^n \tag{1}$$

with values of m from 0.39–0.43 and values of n from 0.33–0.39, depending on the set of data. With this form of correlation, if x/d^* were given and comparisons were made for equal q_j/q_a as M_j was changed, the expression would indicate no change in penetration of the jet centerline. The data shown in Fig. 8 indicate that at constant x/d^* and q_j/q_a, an increase in M_j increased the penetration of the outer boundary of the jet. Unless there is an entirely different behavior between the jet centerline and the outer boundary, the expression of Ref. 8 does not properly describe the behavior of a supersonic jet injected into a supersonic stream.

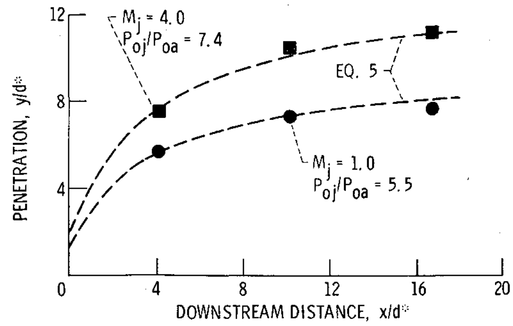

Fig. 10 Comparison of correlation Eq. (5) with data for representative values of Mach number and pressure.

Vranos and Nolan[4] arrived at a correlation equation of the form

$$y/d_e = 1.68[(q_jM_a/q_aM_j)\cos^2\alpha]^{0.5}(x/d_e)^{0.0866} \tag{2}$$

where for normal supersonic injection, d_e is the nozzle exit diameter and $\cos\alpha = 1$. A comparison is made in Fig. 9 between the above expression and the data obtained from a crossplot of Fig. 8. Qualitatively both show a decrease in y/d_e with Mach number; however, quantitatively the agreement was poor. It should be noted that penetration has been previously nondimensionalized by d^* rather than d_e.

Present Correlation

In order to correlate the data of this study, an expression containing the primary variables was assumed

$$y/d^* = C(P_{oj}/P_{oa})^a(M_j)^b(x/d^* + 0.5)^c \tag{3}$$

A constant value of $d^*/2$ was included in the last term so that the calculated position of the 1% He boundary would have a finite vertical displacement at the injection station ($x/d^* = 0$). A multi-variate regression analysis[10] was used to obtain those values of the constants C, a, b, and c, which gave the best least-squares fit of the data. The resulting expression was

$$y/d^* = 1.52(P_{oj}/P_{oa})^{0.567}(M_j)^{0.114}(x/d^* + 0.5)^{0.286} \tag{4}$$

The approximately half-power exponent on the pressure ratio agreed with the pressure dependence noted in Fig. 7.

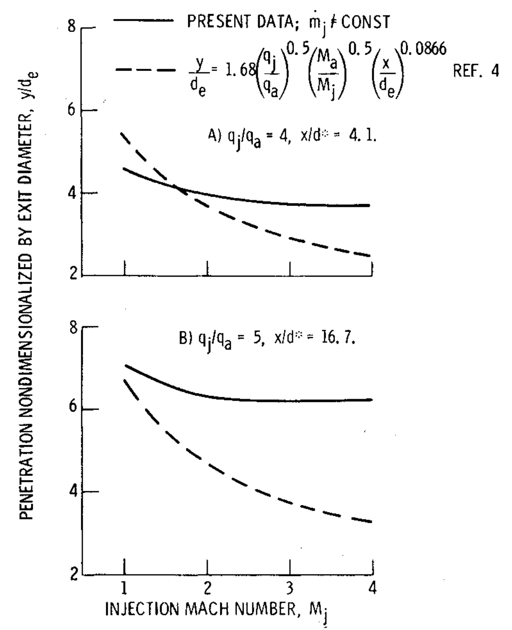

Fig. 9 Effect of injection Mach number on penetration when nondimensionalized by jet exit diameter.

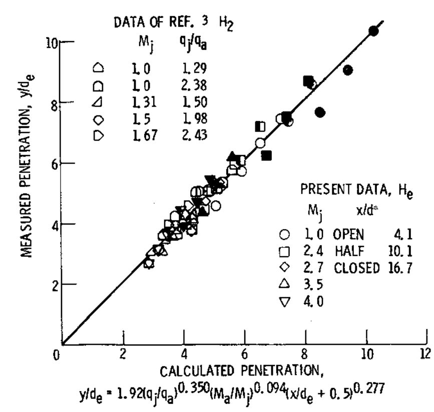

Fig. 11 Comparison of measured penetration and penetration calculated from correlation Eq. (6).

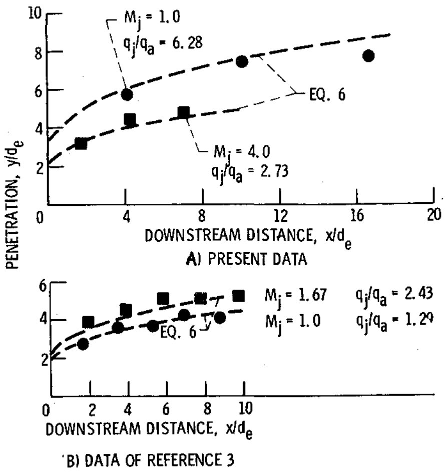

Fig. 12 Comparison of correlation Eq. (6) with data for representative values of Mach number and pressure.

The x/d^* exponent was less than that obtained for a subsonic main stream [Eq. (1)]; and the Mach number exponent indicated a fairly weak dependence on M_j as shown in Figs. 5 and 6. This expression, however, did not produce a very satisfactory trajectory shape when compared to the experimental data, the reason being a y/d^* dependence on x/d^* not amenable to a least-squares fit. To correct this difficulty, y/d^* was normalized by the pressure ratio and Mach number [taken to the powers shown in Eq. (4)], and this expression was plotted against x/d^*. A hyperbolic equation was found to fit the data plotted in this fashion. This expression was used in Eq. (4) to give the final correlation equation for the data of this study:

$$y/d^* = (P_{oj}/P_{oa})^{0.507}(M_j)^{0.114}(x/d^* + 0.5)/(1 + 0.24\, x/d^*) \quad (5)$$

The penetration calculated from this equation and the measured penetration were in very good agreement (±7%). Equation (5) can also be written in terms of momentum ratio through a transformation in terms of γ_j and M_j between pressure and momentum. It should be pointed out here that P_{oa} and M_a were not variables in the experiment and that the correlation may need modification as these parameters are varied.

Comparison was made between the jet boundary trajectory calculated from Eq. (5) and the data of this study for representative values of Mach number and pressure (Fig. 10). Equally good agreement was obtained for all conditions run.

The penetration data of this study have been correlated with the penetration data of Ref. 3. Reference 3 is for the case of sonic and supersonic injection of hydrogen into a Mach 2.72 freestream. The outer edge of the jet was determined from concentration profiles. Equation (5) could not be used to correlate both sets of data since variations in freestream Mach number are not accounted for in this expression. Several correlation forms were attempted using both sets of data in the regression analysis. The most satisfactory expression which has been found to date is one similar to that used by Vranos and Nolan[4]

$$y/d_e = 1.92(q_j/q_a)^{0.350}(M_a/M_j)^{0.094}(x/d_e + 0.5)^{0.277} \quad (6)$$

The comparison of measured and calculated penetration is shown in Fig. 11. Reasonably good agreement was found. The exponent on q_j/q_a, however, was lower than that shown in Fig. 8. It is possible that better agreement was not obtained because of the differences in the ranges of variables used in the two studies (Table 2). Other correlation forms may exist which would incorporate the variables in a different fashion and which would show a more consistent dependence on the variables over wide ranges of conditions. Comparison of the calculated trajectories from Eq. (6) with the present data are shown in Fig. 12a for the same conditions shown in Fig. 10. Trajectory comparisons are also made for two representative cases for the Orth and Funk data (Fig. 12b). Trajectories calculated from Eq. (6) agree well with the Orth and Funk data. There is a slight sacrifice in the agreement with the present data compared to the agreement obtained with Eq. (5) (Fig. 10). Some improvement in the trajectory shapes could be obtained by the same technique used on Eq. (4).

Table 2 Range of variables

	Present study, He	Ref. 3 H_2
M_j	1–4	1–1.67
M_a	2	2.72
P_{oj}/P_{oa}	3.5–9.6	0.78–1.4
q_j/q_a	2.05–10.9	1.29–2.43
x/d^*	4.1–16.7	1.7–11.2

Summary of Results

Helium was injected from a flat plate into a supersonic airstream. Injection Mach number M_j, total pressure P_{oj}, and downstream measuring station x/d^* were varied. The effect of M_j on penetration was determined from concentration measurements.

Penetration increased with increasing M_j. The dependence of penetration on M_j increased with downstream distance. As much as a 25% increase in penetration was found when M_j was increased from sonic to Mach 4. Much of the observed increase in penetration was found to occur between Mach 1 and Mach 2.5. No maxima in jet penetration with M_j were observed.

The data of this study and those of a report in the literature were correlated with an expression containing momentum, Mach number, and downstream distance.

Normalizing the jet penetration by mass flow correlated the data for any given downstream position only.

The data varied with jet to freestream momentum ratio to the one-half power but only for a given Mach number and downstream position. Several correlations in the literature based on momentum ratio did not adequately describe the present data.

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